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Diagnostic of spectral lines in magnetized solar atmosphere: Formation of the Hβ line in sunspots

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Formation of the Hβ λ4861.34 Å line is an important topic related to the diagnosis of the basic configuration of magnetic fields in the solar and stellar chromospheres. Specifically, broadening of the Hβ λ4861.34 Å line occurs due to the magnetic and micro-electric fields in the solar atmosphere. The formation of Hβ in the model umbral atmosphere is presented based on the assumption of non-local thermodynamic equilibrium. It is found that the model umbral chromosphere is transparent to the Stokes parameters of the Hβ line, which implies that the observed signals of magnetic fields at sunspot umbrae via the Hβ line originate from the deep solar atmosphere, where $\lg\tau_c \approx -1$ (about 300 km in the photospheric layer for our calculations). This is in contrast to the observed Stokes signals from non-sunspot areas, which are thought to primarily form in the solar chromosphere.

solar spectrum, polarization-Stokes parameters, magnetic fields

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1 Introduction

With the discovery of the Zeeman effect by Zeeman [1], scientists can therefore extract information on magnetic fields in remote objects through spectro-polarimetry. Hale [2] was the first to make use of this principle in astrophysics, which led him to find pronounced Zeeman splitting in sunspots. Quantitative measurements of solar magnetic fields were made possible. Since the 1980s, a modern solar vector magnetograph (Solar Magnetic Field Telescope) was developed in China [3]. It was the inauguration of the systematic observations and the corresponding theoretical research of solar magnetic fields in China.

Chromosphere is a very important layer in the diagnostic of solar and stellar activities [4]. The Hβ λ4861.34 Å line has been used at Huairou Solar Observing Station (HSOS) of National Astronomical Observatories of the Chinese Academy of Sciences for the measurements of the solar chromospheric magnetic fields [5]. Similar magnetograms were obtained at other observatories, such as Crimea and Kitt Peak [6-8]. These observations have proved useful to enhance our understanding of the different properties of solar active phenomena caused by magnetic fields [9].

Diagnostic on the formation layers of spectral lines in solar or stellar atmospheres is a basic topic, especially for the measurements of the magnetic fields with Stokes parameters of spectral lines. In the following, we present the analysis on the formation of the Hβ λ4861.34 Å line in the magnetic fields of solar atmosphere, especially in the sunspot umbral atmosphere.

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2 Solar model atmospheres

The Hβ line forms in a relatively wide solar atmosphere. This is related to the basic states on the statistical distribution of energy level populations of hydrogen atoms in the solar atmosphere. To study the formation of the Hβ line in the magnetized solar atmosphere, the non-local thermodynamic equilibrium (NLTE) population departure coefficients b_i are defined as:

$$b_\mathrm{l} = N_\mathrm{l}/N_\mathrm{l}^\mathrm{LTE}, \quad b_\mathrm{u} = N_\mathrm{u}/N_\mathrm{u}^\mathrm{LTE}, \tag{1}$$

where $N_{(\mathrm{l,u})}$ is the actual population and $N_{(\mathrm{l,u})}^\mathrm{LTE}$ the Saha-Boltzmann values for the lower (l) and upper (u) levels. The treatment of NLTE is suitable for analyzing the formation of spectral lines in the upper solar atmosphere due to the weakening contribution of particle collisions, while the approximation of local thermodynamic equilibrium (LTE) is normally acceptable in the lower solar atmosphere due to the high density of the particle populations, where the departure coefficients are $b_i \approx 1$ in eq. (1).

Figure 1(a) shows the atmospheric model of the quiet Sun with the distributions of the departure coefficients b_2 and b_4 of the hydrogen atomic levels $n=2$ and $n=4$ at different continuum optical depths (Vernazza et al. [10]). It is found that the departure coefficients b_i in the lower solar atmosphere are of the order unity, and they increase drastically as hydrogen density N_h and electron density N_e decrease, and as the temperature increases at low optical depths. The departure coefficients tend to decrease at very thin depths.

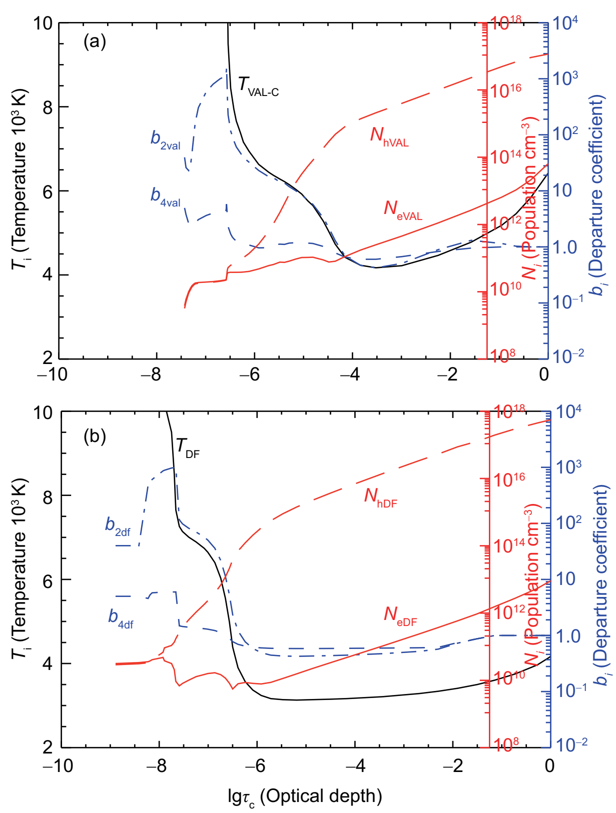

Figure 1 (Color online) Distribution of T, N_h, N_e, and the departure coefficients (b_2 and b_4) of the hydrogen levels. The continuum optical depth is displayed in $\lg\tau_\mathrm{c}$ scale in the atmospheric model of the quiet Sun (a) by Vernazza, Avrett and Loeser (VAL, 1981) [10] and the sunspot umbra model (b) by Ding and Fang (DF, 1991) [11].

A sunspot umbral model with the distributions of temperature T, hydrogen density N_h and electron density N_e in Figure 1(b) was provided by Ding and Fang [11].

Assuming that the same thermodynamic mechanism operates both in the quiet Sun and sunspot umbrae models, the departure coefficients b_i of the hydroden atomic levels in the latter atmosphere model can be phenomenologically provided by using the tendencies of the departure coefficients b_i found for the quiet Sun as a proxy. Figure 1(b) shows the population departure coefficient $b_i \approx 1$ in the deep umbral model atmosphere, which increases with height in the interval $\lg\tau = [-6, -7.5]$ and decreases gradually with height in the range $\lg\tau = [-7.5, -9]$.

In thermodynamic equilibrium, atoms are distributed over their bound levels according to the Boltzmann excitation equation [12]. The continuum normally forms in the lower solar atmosphere, thus the Planck function $B(T,\nu)$ can be used as its source function

$$B^C(T,\nu) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}, \tag{2}$$

and the source function of Hβ line is

$$S^L(\mathrm{H}_\beta) = \frac{2h\nu^3}{c^2}\frac{1}{\frac{b_2}{b_4}\exp(h\nu/kT)-1}, \tag{3}$$

where the values of b_2 and b_4 are shown in Figure 1. The self-consistency of these departure coefficients of the Hβ line with the observations can be found in the following sections.

3 Hydrogen lines, magnetic fields and ionized solar atmosphere

To analyze the contribution of the magnetic and micro-electric fields to the broadening of hydrogen lines in the solar atmosphere, the Hamiltonian can be written in the following condensed form:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_\mathrm{Z} + \mathcal{H}_\mathrm{S}, \tag{4}$$

where $\mathcal{H}_0$ includes the Hamiltonian $\mathcal{H}_\mathrm{NR}$, $\mathcal{H}_\mathrm{fine}$ and $\mathcal{H}_\mathrm{hyper}$, and $\mathcal{H}_Z = \mu_0\,\mathbf{B}_e\cdot(\mathbf{L}+2\mathbf{S})$ and $\mathcal{H}_S = e_0\,\mathbf{E}_e\cdot\mathbf{r}$. We have purposely chosen the Hamiltonian $\mathcal{H}_Z$ and $\mathcal{H}_S$ of the Zeeman and Stark effects to analyze the broadening of hydrogen lines (such as, the Hβ line) in the solar atmosphere, as magnetic and micro-electric fields commonly contribute to these lines. The general results for the fine structure of hydrogen lines can be found in the monograph of Bethe and Salpeter [13].

Next, we will consider the hydrogen atoms and perturbing, charged particles in external magnetic and electric fields (eq. (4)). The perturbation equation is

$$\mathcal{H}_0 + \mathcal{H}_Z + \mathcal{H}_S \mid \psi\rangle = (E_0 + E') \mid \psi\rangle, \quad (5)$$

where E_0 and E' are the energy's eigenvalue and perturbation, respectively. One can calculate the shifts $\triangle\lambda_j$ of the sub-components of the spectral line by means of eq. (5) when the magnetic and electric fields are considered.

Left-multiply eq. (5) by $\langle\psi|$, and use the identity for the perturbing Hamiltonian

$$\langle n, l, j, m_j \mid \mathcal{H}_Z + \mathcal{H}_S \mid n, l', j', m'_j\rangle \equiv K_{q,q'}, \quad (6)$$

the suffixes q and q' in K refer to various possible wave function states. Assume the perturbed wave function to be a linear combination of unperturbed wave functions:

$$\mid \psi\rangle = \sum_{q'=1}^{2n^2} c_{q'} \mid \psi_{q'}\rangle, \quad (7)$$

eq. (6) then becomes

$$\sum_{q'=1}^{2n^2} (K_{q,q'} - E'\delta_{qq'})c_{q'} = 0, \quad (8)$$

where $q = 1, 2, \cdots, 2n^2$. Normally, the shifts of the energy levels depend on the distribution of the magnetic and electric field nearby the hydrogen atom.

Normally, the general solution of the eigenvalue problem for hydrogen lines relates to the direction of the magnetic field and microscopic electric field. By means of eq. (8), we can find the $2n^2$ possible splits of the upper and lower energy levels and the corresponding $2n^2$ probabilities of the wave functions in various directions and intensities of the electric field for each sub-level of the hydrogen line. Then, we can calculate the shifts $\triangle\lambda_j$ of the sub-components j of the spectral line [5]. The calculation of the Hermitian matrix of the perturbing Hamiltonian was presented also by Casini and Landi Degl'Innocenti [14].

As the microscopic electric field in the solar atmosphere is statistically isotropic, we assumed here that the polarized states of the sub-components of the Hβ line still rely on the magnetic field. Because the perturbed wave function is a linear combination of unperturbed wave functions, the polarized states of the sub-components of the line are also related to the magnetic quantum number m, e.g., $\triangle m = 0$ for the π component and $\triangle m = \pm 1$ for both σ components.

The observed profiles of the Hβ λ4861.34 Å line in the quiet Sun and sunspot umbra with the overlap of some photospheric lines in the wings are shown in Figure 2. In the following, we also neglect the asymmetry of the spectral line, which is normally believed to be caused by the gradient of the line of sight velocity field in the solar atmosphere [15].

4 Formation of the Hβ line in solar magnetic atmosphere

The numerical Stokes profiles (I, Q, U, V) of the Hβ line can be calculated by means of the Unno-Rachkovsky equations of polarized radiative transfer of spectral lines (Unno [16] and Rachkovsky [17, 18]):

$$\mu \frac{\mathrm{d}}{\mathrm{d}\tau_\mathrm{c}} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_0 + \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_0 + \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_0 + \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_0 + \eta_I \end{pmatrix} \begin{pmatrix} I - S \\ Q \\ U \\ V \end{pmatrix}, \quad (9)$$

where the symbols have their usual meanings as given by Landi Degl'Innocenti [19], $\mathrm{d}\tau_\mathrm{c} = -\kappa_c \mathrm{d}s$, η_I, η_Q, η_U, η_V are the Stokes absorption coefficient parameters and ρ_Q, ρ_U and ρ_V are related to magneto-optical effects, S represents the source functions related to eq. (2) for the continuum and eq. (3) for the Hβ line.

By comparing with the formulae of the non-polarized hydrogen line proposed by Zelenka [20], the contributions of the statistically distributed microscopic electric field with the Zeeman effects in eqs. (6)-(8) rising from the magnetic field in the solar atmosphere are considered in the absorption matrix of eq. (9) for the Hβ line (see refs. [5, 21]).

It is assumed here that, when the degeneracy of an energy level disappears under the actions of magnetic and microscopic electric fields, each magnetic energy level keeps its

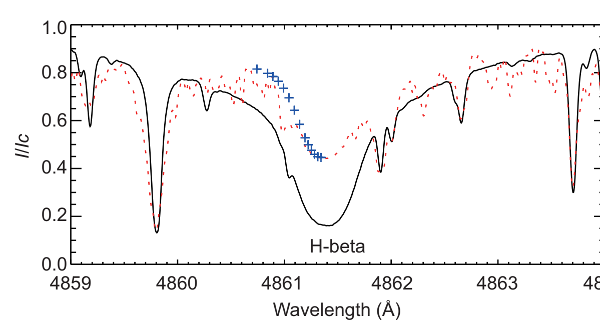

Figure 2 (Color online) Hβ line profiles of the quiet Sun (solid line) and of the sunspot umbra (red dotted line) observed by the National Solar Observatory of USA (L. Wallace, K. Hinkle, and W. C. Livingston, http://diglib.nso.edu/ftp.html). The blue plus symbols mark the results of the numerical calculation.

original departure coefficient.

In Figure 3, it is noticed that the numerically calculated residual intensity at the core of the Stokes I profile is of the order 0.5 in the umbral model atmosphere, and the profile of Stokes I is roughly consistent with the observed umbral one in Figure 2, if the blended lines are ignored (The calculated result is also marked by plus symbols in the blue wing of the line in Figure 2). Note that it is weaker than that of the quiet Sun. There are no obvious differences in the Stokes I values between the umbral magnetic field strengths of 1000 and 3000 G. The amplitude of the Stokes V is of the order 10^{-2}, and those of Stokes Q and U are 10^{-4}. These values are similar to those of the quiet Sun (see Figure 11 of ref. [21]). This means measuring the transverse components of the magnetic fields of sunspots is still a technological challenge due to their very weak signals.

Moreover, the peak values of Stokes V are located at 0.2 Å from the line center as calculated using the umbral model atmosphere in Figure 3, and at 0.3 Å for the quiet Sun (see Figure 11 of ref. [21]). Similar tendencies for Stokes Q and U can be found as well. This reflects that the contribution of the Holtsmark broadening in the umbral atmosphere, which depends on the density of the charged particles, is relative weaker than that in the quiet Sun. This result is consistent with the abundance of the electrons N_e in both models.

The key is to know where in the solar atmosphere a given spectral line is formed, which can be used to estimate the possible formation height of the observed magnetogram. This information is provided by the contribution function $C_{\mathbf{I}}$, such as defined by Jin [22] and Stenflo [23], for the Stokes vector $\mathbf{I} = (I, Q, U, V)$ [24]:

$$\mathbf{I}(0) = \int_0^{\infty} C_{\mathbf{I}}(\tau_c)\mathrm{d}\tau_c = \int_{-\infty}^{\infty} C_{\mathbf{I}}(x)\mathrm{d}x, \quad (10)$$

where $C_{\mathbf{I}}(x) = (\ln 10)\tau_c C_{\mathbf{I}}(\tau_c)$ and $x = \lg\tau_c$, τ_c is the continuum optical depth at 5000 Å. Eq. (10) provides the contribution to the emergent Stokes parameters from the different layers of the solar atmosphere with the equivalent source functions $S^{\star}_{\mathbf{I}}$ [25].

The contribution functions of the Hβ line in the magnetic

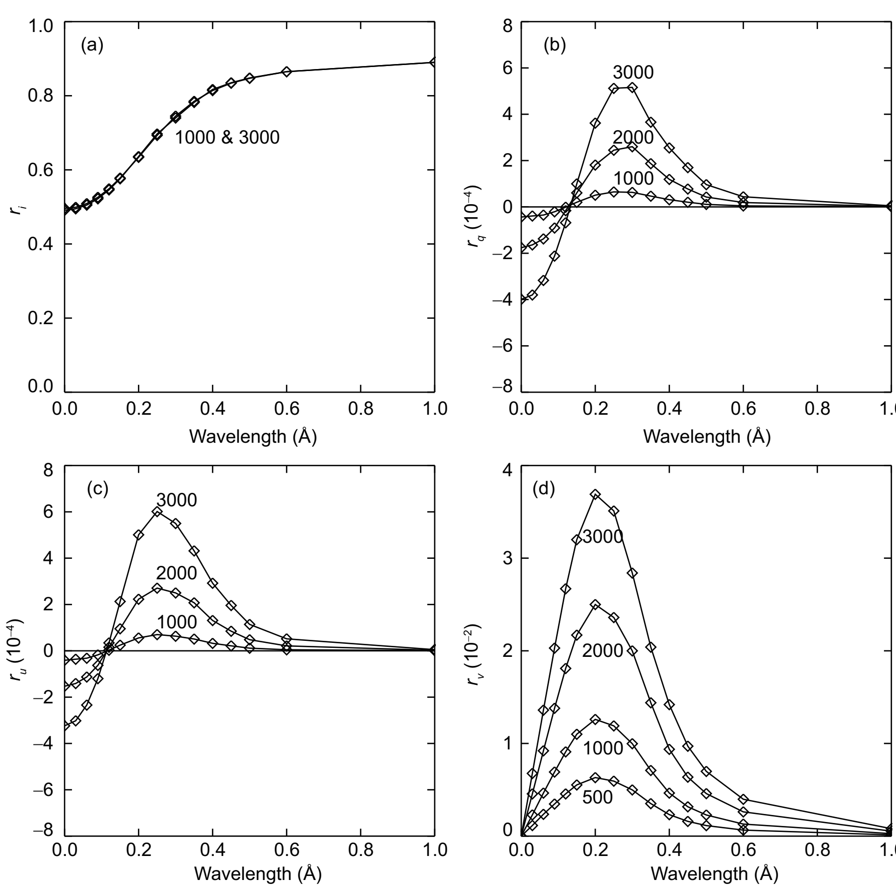

Figure 3 Profiles of Stokes parameters (a) r_i, (b) r_q, (c) r_u, and (d) r_v (i.e. I/I_c, Q/I_c, U/I_c and V/I_c) of the Hβ line calculated for the umbral atmospheric model by Ding & Fang [11]. The homogeneous magnetic field intensity of 500, 1000, 2000, 3000 G for an inclination of ψ=30°, an azimuth of φ=22.5° and $\mu = 1$. I_c is the continuum.

atmospheric model of the quiet Sun were presented by Zhang et al. [21, 26]. The emergent Stokes parameters at the Hβ line center in the quiet Sun almost form in the middle chromosphere (1500-1600 km), and those in the blue wing, −0.45 Å from the Hβ line center, form in the photosphere (300 km) [5].

Figure 4 shows the contributions of the Stokes parameter I, Q, U, and V in the umbra model at 0.0, 0.06, 0.15, 0.25, and 0.45 Å from the center of the Hβ line. It is noticed that the contributions of the emergent Stokes parameters in the higher solar atmosphere (the continuum optical depth $\lg\tau_c \approx -7$) are much weaker than that in the lower atmosphere ($\lg\tau_c \approx -1$), even though there are some differences in the Stokes parameters at different wavelengths from the center of the Hβ line. This result implies that the upper umbral atmosphere is transparent to the Hβ line. The signals of the emergent Stokes parameters of the Hβ line in the umbrae formed mainly in the lower solar atmosphere, while those near the line center in the quiet Sun form in the chromosphere (see Figure 12 of ref. [21]). This result can also be found by comparing the residual intensities at the center of the Hβ line for the quiet Sun and the umbra in Figure 2 due to different absorptions of the line.

A similar observational result of the sunspots in the white light has been named as the Wilson effect. Bray and Loughhead [27] contended that the true explanation of the Wilson effect lies in the higher transparency of the spot material compared to the photosphere. The morphological evidence can be found in Figure 13 of ref. [21]. Similar patterns of a sunspot umbra occur both in the photospheric and the Hβ filtergrams, even when the bright flare ribbons in Hβ are attached to the umbrae in the Hβ filtergram. The blended lines from the deep layer cause the reversed signals in the umbral areas of the Hβ magnetograms as indicated by refs. [21, 28].

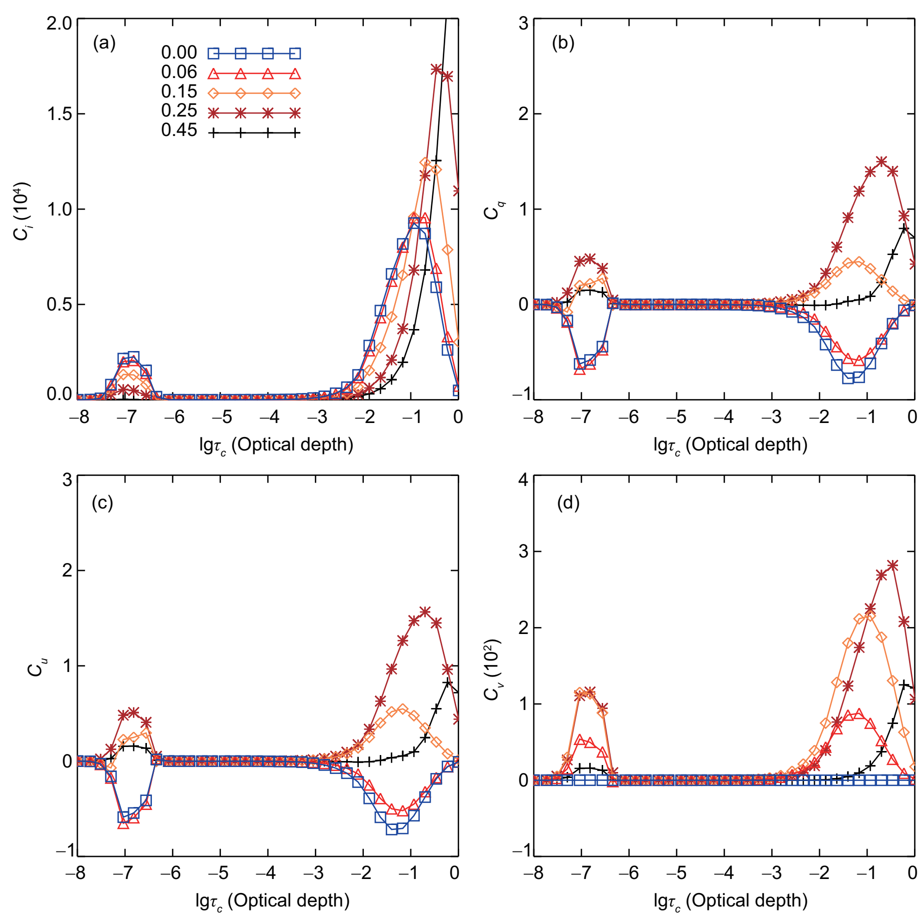

Figure 4 (Color online) Contribution functions C_i (a), C_q (b), C_u (c), and C_v (d) of Stokes parameters I, Q, U, and V of the numerically calculated Hβ λ4861.34 Å line for the umbral atmospheric model by Ding and Fang [11] at wavelengths of $\triangle\lambda$=0.45 (pluses), 0.25 (asterisks), 0.15 (diamonds), 0.06 (triangles) and 0.0 (squares) Å from the Hβ line center. B=1000 G, ψ=30°, φ=22.5° and μ = 1. τ_c is continuum optical depth at 5000 Å. The horizontal coordinates are in logarithmic scale.

A simple schematic sketch of the formation heights of the Stokes parameters observed by the Hβ line is proposed in Figure 5. The red thick dashed line shows the rough detectable heights in the Hβ magnetograms. In the umbrae, the observed Stokes parameters mainly form in deep layers in Figure 4, although weak contributions also arise from the chromosphere in Figure 5. This means that the Hβ line in the sunspot umbrae forms at the continuum optical depth $\lg \tau_c \approx -1$ for our calculations, i.e., about 300 km in the photospheric layer.

5 Discussions and summary

In this paper, the departure coefficients of the hydrogen atom in the model umbral atmosphere under the NLTE hypothesis were presented and compared with their distributions in the quiet Sun. The locations of the peak values of the Stokes *V* of the Hβ line are close to the line center in the model umbral atmosphere as compared to those of the quiet Sun. This result is consistent with the weaker contribution of the micro-electric field to broadening the Hβ line in the solar umbral atmosphere than in the quiet Sun.

It is worth noting that, because the sunspot umbra exhibits lower temperature characteristics compared to the quiet Sun, the umbral chromosphere is generally transparent to the spectral lines that require higher temperature excitation, such as the Hβ line. Moreover, the ratio of the emergent Stokes parameters of spectral lines from different depths of the solar atmosphere depends on the parameters of the magnetic and atmospheric models used in the radiative transfer equations. In order to verify it, accurate observations are still necessary and important.

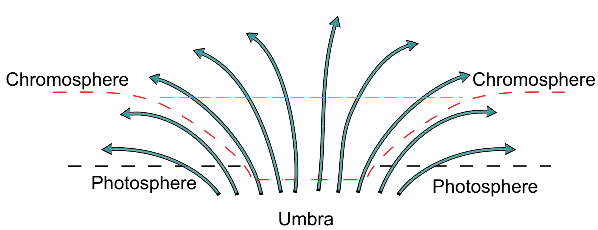

Figure 5 (Color online) A schematic sketch of the estimated formation height of the measured Stokes parameters of the Hβ line for sunspot regions. The red thick dashed line marks the major contribution heights of the Stokes parameters of the Hβ line, while the orange thin dashed line shows the weak contribution of the line in the chromosphere.

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