# NOTES

# A parametric method of retrieving ocean wave spectra from synthetic aperture radar images

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**Abstract** A parametric method that extracts the ocean wave directional spectra from synthetic aperture radar (SAR) image is presented. The 180° ambiguity of SAR image and the loss of information beyond the azimuthal cutoff can be overcome with this method. The ocean wave spectra can be obtained from SAR image directly by using iteration inversion mapping method with forward nonlinear mapping. Some numerical experiments have been made by using ERS-1 satellite SAR imagette data. The ocean wave direction retrieved from SAR imagette data is in agreement with the wind direction from the scatterometer data.

**Keywords:** SAR, directional ocean wave spectrum, ocean wave remote sensing.

OCEAN wave generally refers to wind wave and swell. Systematical study of ocean wave spectrum can make people well understand the formation, mechanism, inner structure and characteristics of ocean wave, which is of great importance for ocean engineering, ocean transportation as well as theoretical researches of ocean wave. The directional ocean wave spectrum has been proved extremely difficult to measure by conventional methods, and it is nearly impossible to measure it in large area. Remote sensing technology is a promising method to solve the problem. High-resolution SAR image has shown the potential of measuring the directional ocean wave spectrum. The ocean wave directional spectrum cannot be obtained directly from SAR image although a great amount of work has been done[1—3].

Up to now, there are two kinds of methods to obtain ocean wave directional spectrum from SAR image. One is the linear transfer function method[1,3,8,11]; however, there is 180° directional ambiguity, and the other is the iteration inversion mapping method with forward nonlinear mapping[6,7,9]. The 180°

directional ambiguity can be removed by using the first guess spectrum for the iteration inversion mapping method, and ocean wave directional spectrum is very sensitive to it. Therefore, it is very important to give a good first guess spectrum. Both of the above problems are caused by the symmetry of SAR image spectrum. The ocean wave spectrum could not be obtained by the above methods if other conditions were not available. A new method, a parametric method, is presented here and can overcome the present difficulties. The ocean wave spectrum may be obtained directly from a SAR image spectrum.

## 1 A parametric method

SAR ocean wave imaging mechanism has three basic modulation processes: ( i ) tilt modulation, which is the change in the local incidence angle of the facet through the long wave slope. The backscattering cross section is the biggest when the ocean wave moves towards the radar, and the smallest away from the radar. The tilt modulation depends on the ripple wave spectrum and radar incident angle; ( ii ) the hydrodynamic modulation interaction between ripple wave and long wave, which modulates the energy and wave number of the short Bragg scattering ripple wave; ( iii ) motion effects, which include velocity bunching and smearing, producing a Doppler shift in the return signal and inducing an azimuthal displacement of the scattering element in the image plane. If the long wave peak is perpendicular to the radar velocity, the displacement of the scattering element in front of the wave peak will move towards the azimuthal direction, and the displacement of the scattering element behind the wave peak will move backwards. The azimuthal displacement is only less than one ocean wave length if the long wave amplitude is not so big. Even if the first two kinds of modulations are ignored, the image is black near the ocean peak and light near wave trough. This phenomenon is called velocity bunching. If the amplitude of the long wave is too big, the azimuthal displacement will be longer than one ocean wavelength and the velocity bunching modulation will be strong nonlinear. The tilt modulation and hydrodynamic modulation are the linear processes, but the velocity bunching is a nonlinear one.

In the framework of linear modulation theory, the surface elevation $\zeta$ and the variations of the local backscattering cross section $\sigma(r,t)$ sensed by a real aperture radar (RAR) may be represented as a superposition of propagating wave components,

$$\zeta(r,t) = \sum_k \zeta_k \exp(\mathrm{i}(k \cdot r - \omega t)) = \mathrm{c.c.}, \tag{1}$$

$$\sigma(r,t) = \bar{\sigma}\left\{1 + \left[\sum_k T_k^{\mathrm{R}} \zeta_k \exp(\mathrm{i}(k \cdot r - \omega t)) + \mathrm{c.c.}\right]\right\}, \tag{2}$$

where $\bar{\sigma}$ denotes the spatially averaged specific cross section, $T_k^{\mathrm{R}}$ is the RAR modulation transfer function, including tilt modulation $T_k^{\mathrm{t}}$ and hydrodynamic modulation $T_k^{\mathrm{h}}$. For the ERS-1 satellite SAR, they can be presented as follows[7]:

$$T_k^{\mathrm{t}}(k) = 4\mathrm{i}k_y \cot\theta(1 + \sin^2\theta)^{-1}, \tag{3}$$

$$T_k^{\mathrm{h}} = 4.5\left(\frac{\omega - \mathrm{i}\mu}{\omega^2 + \mu^2}\right)k\omega\left(\frac{k_y^2}{k^2} + Y\right). \tag{4}$$

For the RAR, the image variance spectra and ocean wave spectra are defined by

$$\langle I^{\mathrm{R}^2}\rangle = \sum_k P_k^{\mathrm{R}} = \sum_k \langle I_k^{\mathrm{R}^*} I_k^{\mathrm{R}}\rangle,$$
$$\langle \zeta^2 \rangle = \sum_k F_k = 2\sum_k \langle \zeta_k^* \zeta_k \rangle. \tag{5}$$

Then the linear relation between RAR image variance spectra and ocean wave spectra is

$$P_k^{\mathrm{R}} = \frac{1}{2}\left(\left|T_k^{\mathrm{t}} + T_k^{\mathrm{h}}\right|^2 F_k + \left|T_{-k}^{\mathrm{t}} + T_{-k}^{\mathrm{h}}\right|^2 F_{-k}\right). \tag{6}$$

With the velocity bunching model, the relation between the SAR and RAR images is given by

$$I^{\mathrm{S}}(r) = \int I^{\mathrm{R}}(r')\delta(r - r' - x(r'))\mathrm{d}r', \tag{7}$$

where $x(r')$ is the azimuthal displacement of the apparent position of a backscattering element in the image plane. The relation between an ocean wave spectrum $F(k)$ and an SAR image spectrum $P^{\mathrm{s}}(k)$ is given by a closed integral transform after some straightforward algebra[7],

$$P^{S}(k)=(2\pi)^{-1}\exp(-k_x^2\xi^2)\int d\bar{r}e^{-i\bar{k}\cdot\bar{r}}\exp[k_x^2\beta^2 f^{v}(\bar{r})]\cdot$$
$$\{1+f^{R}(\bar{r})+i\,k_x\beta[f^{RV}(\bar{r})-f^{RV}(-\bar{r})]$$
$$+(k_x\beta)^2[f^{RV}(\bar{r})-f^{RV}(0)][f^{RV}(-\bar{r})-f^{RV}(0)]\},\tag{8}$$

where

$$P^{S}(\bar{k})=P_k^{S}/\Delta\bar{k},\tag{9}$$

$$F(\bar{k})=F_k/\Delta\bar{k},\tag{10}$$

$$f^{R}(\bar{r})=\langle I^{R}(\bar{r}+\bar{x})I^{R}(\bar{r})\rangle=\frac{1}{2}\int\left\{F(\bar{k})\,|T_k^{R}|^2+F(-\bar{k})\,|T_{-k}^{R}|^2\right\}e^{i\bar{k}\cdot\bar{r}}d\bar{k},\tag{11}$$

$$f^{V}(\bar{r})=\langle V(\bar{r}+\bar{x})V(\bar{r})\rangle=\frac{1}{2}\int\left\{F(\bar{k})\,|T_k^{V}|^2+F(-\bar{k})\,|T_{-k}^{V}|^2\right\}e^{i\bar{k}\cdot\bar{r}}d\bar{k},\tag{12}$$

$$f^{RV}(\bar{r})=\langle I^{R}(\bar{r}+\bar{x})V(\bar{r})\rangle=\frac{1}{2}\int\{F(\bar{k})T_k^{R}(T_k^{v})^{*}+F(-\bar{k})(T_{-k}^{R})^{*}(T_{-k}^{v})\}e^{i\bar{k}\cdot\bar{r}}d\bar{k},\tag{13}$$

$$\xi^2=\beta^2\int F(\bar{k})\,|T_k^{V}|^2 d\bar{k}.\tag{14}$$

It is impossiple to get the ocean wave spectrum directly from eq. (8) because it is a nonlinear function. A parametric method is used here. Suppose that the parametric ocean wave spectrum is[14]

$$F(\omega,\theta)=\frac{1}{2}\Phi(\omega)\beta\mathrm{sech}^2\beta\{\theta-\theta(\omega)\},\tag{15}$$

where $\bar{\theta}$ is the average ocean wave direction and

$$\beta=\begin{cases}2.61(\omega/\omega_p)^{-1.3}, & 0.56<\omega/\omega_p<0.95,\\ 2.28(\omega/\omega_p)^{-1.3}, & 0.95<\omega/\omega_p<1.6,\\ 1.24, & \text{otherwise},\end{cases}\tag{16}$$

$$\Phi(\omega)=\alpha g^2\omega^{-5}(\omega/\omega_p)\exp\left\{-\left(\frac{\omega_p}{\omega}\right)^4\right\}\gamma^{\Gamma},\tag{17}$$

$$\alpha=0.006(U_c/c_p)^{0.55},\ 0.83<U_c/c_p<5,\tag{18}$$

$$\gamma=\begin{cases}1.7, & 0.83<U_c/c_p<1,\\ 1.7+6.0\lg(U_c/c_p), & 1\leqslant U_c/c_p<5,\end{cases}\tag{19}$$

$$\Gamma=\exp\{-(\omega-\omega_p)^2/2\sigma^2\omega_p^2\},\tag{20}$$

$$\sigma=0.08[1+4/(U_c/c_p)^3],\ 0.83<U_c/c_p<5.\tag{21}$$

We choose $\alpha,\omega_p,\bar{\theta}$ as the parameters to be determined. It is obvious that it is impossible to determine these three parameters from expression (8). The SAR image spectrum has 180° directional ambiguity because the SAR image spectrum is obtained by the Fourier transform of SAR image. In order to reject the 180° directional ambiguity, another condition must be provided. In the past, the first guess ocean wave spectrum was used to reject the 180° directional ambiguity. However, the result greatly depends on the first guess spectrum[7]. We solve the problem by choosing two close SAR sub-images with different incident angles in the range direction from one SAR imagette data and suppose that the ocean wave spectra of the two ocean areas corresponding to the two SAR sub-images are the same. In fact, the supposed condition tallies with the facts. Therefore, we define the cost function

$$J=\sum(P_1(k)-\hat{P}_1(k))^2\hat{P}_1(k)+(P_2(k)-\hat{P}_2(k))^2\hat{P}_2(k),\tag{22}$$

where $P_1(k),\hat{P}_1(k),P_2(k),\hat{P}_2(k)$ are the simulated SAR spectra and the observed spectra at different incident angles, respectively. $J(\alpha,\omega_p,\bar{\theta})$ is a very complex function. The optimal fit wave spectral parameters $\alpha$, $\omega_p$, $\bar{\theta}$ can be obtained by using the grid optimum method that minimizes the cost function $J$.

## 2 Numerical results and analysis

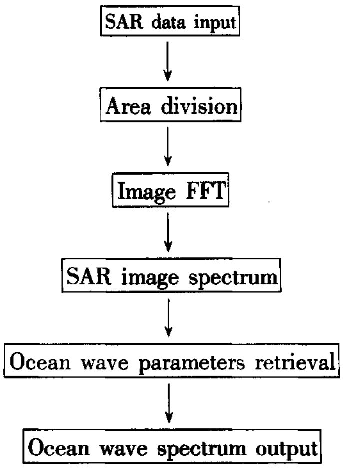
The ocean wave spectrum is obtained by the following three steps.

( i ) The size of ERS-1 SAR imagette data used below is 600 pixels in range direction, 320 pixels in azimuthal direction. The range spacing is 20 m, and the azimuthal spacing 16 m. We choose two 256 × 256 pixel SAR image blocks in the range direction. Each image block is subdivided into four 128 × 128 pixel sub-images in order to reduce the noise.

( ii ) SAR image spectra. The spectrum of every sub-image is obtained with Fourier transform. The spectrum of each block image is an average of the spectra of four sub-images.

( iii ) Inversion with optimal method. The optimal fit wave spectral parameters can be obtained by using the grid optimum method that minimizes the cost function $J$. The ocean wave spectrum is given by substituting the parameter into (15).

The chart to retrieve an ocean wave spectrum from a SAR image spectrum is as follows:




The two different orbit SAR imagette data on 17 Nov. 1993 are chosen. The information of the images is shown in table 1.

Table 1 The information of the SAR image

| Example | Time | Satellite flight direction | Central longitude | Central latitude |
|---|---|---|---|---|
| 1 | 1993-11-17 21:49:37.424 | 339.782° | 358.322° | 62.864° |
| 2 | 1993-11-17 12:00:19.485 | 201.526° | 350.464° | 65.011° |

The derived ocean wave directional spectrum is shown in fig. 1. The ocean wave direction and the wind direction measured by satellite scatterometer are similar (fig. 2). The best fit SAR spectrum agrees well with the observed SAR spectrum.

The position of the other SAR image with a descending orbit is near the icelands. The wind direction measured by scatterometer is west by north (fig. 4), and the ocean wave direction retrieved from SAR image is westward (fig. 3). The 180° direction ambiguity is overcome successfully by the present method and the best fit SAR spectrum is also in good agreement with the observed SAR spectrum for the single peak spectrum. However, it is not so good for multi-peak spectrum because of the supposition that the parametric ocean wave spectrum is a single peak one. In addition, the simulated ocean wave spectrum is overestimated for the light wind, leading to some limitations of the present method. We can only compare our results with the scatterometer data because of lack of the *in-situ* data. Therefore, some validation and further research need to be done in the future.

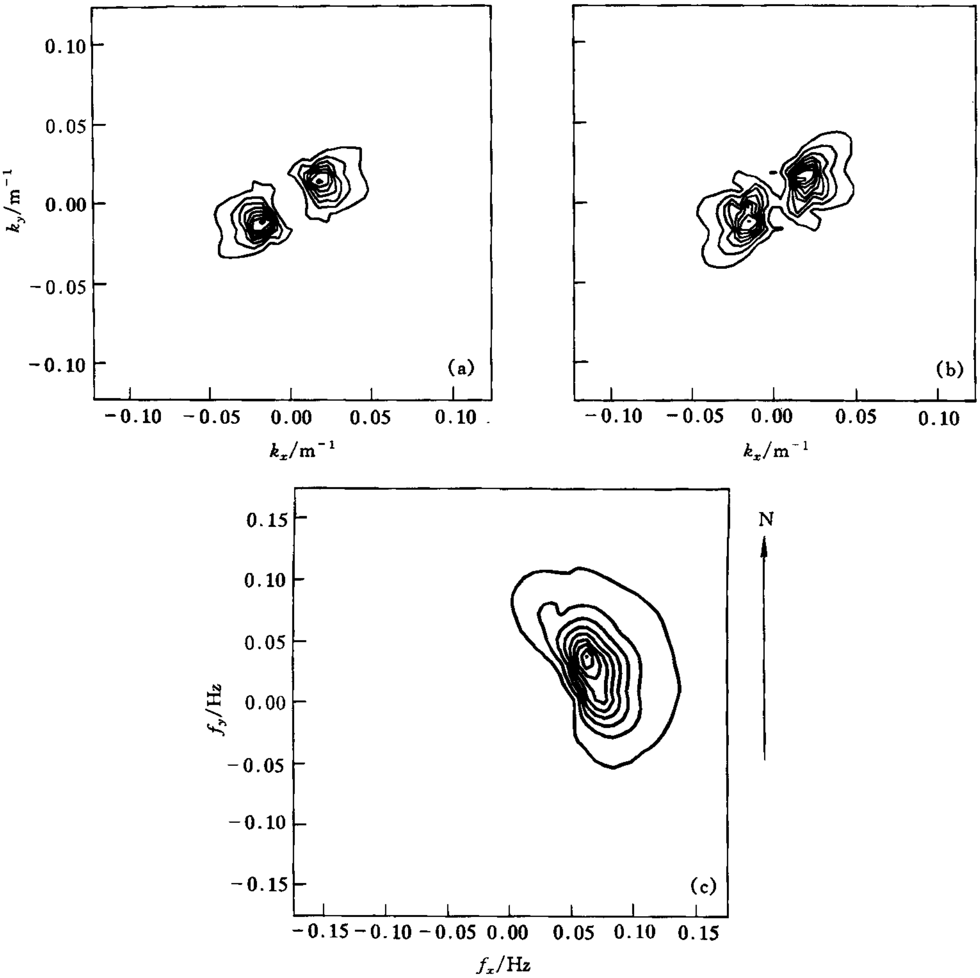



Fig. 1. (a) Stimulation SAR image spectrum; (b) observed SAR image spectrum; (c) inversion of ERS-1 SAR image spectrum.

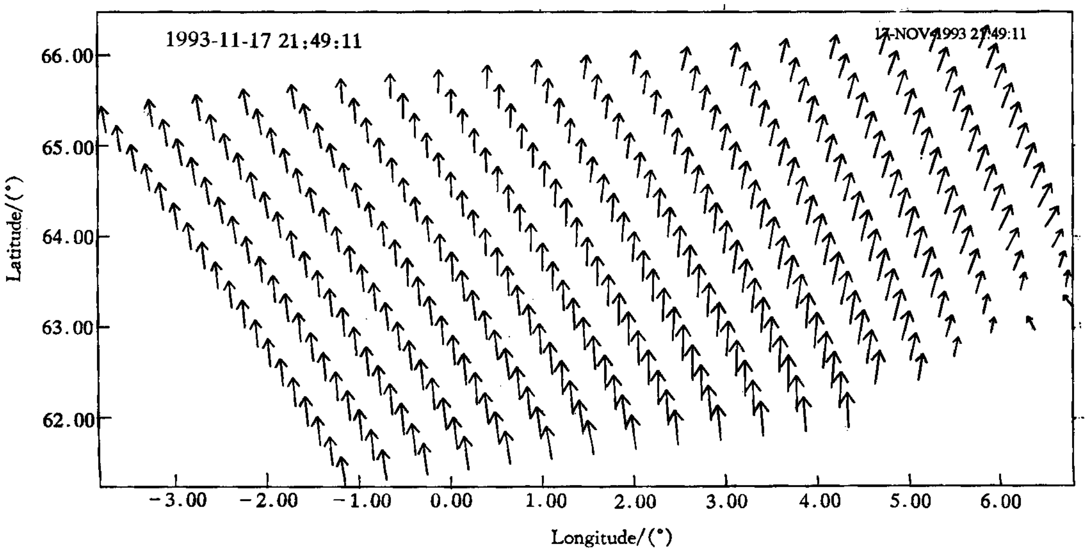



Fig. 2. Wind field derived from ERS-1 scatterometer.

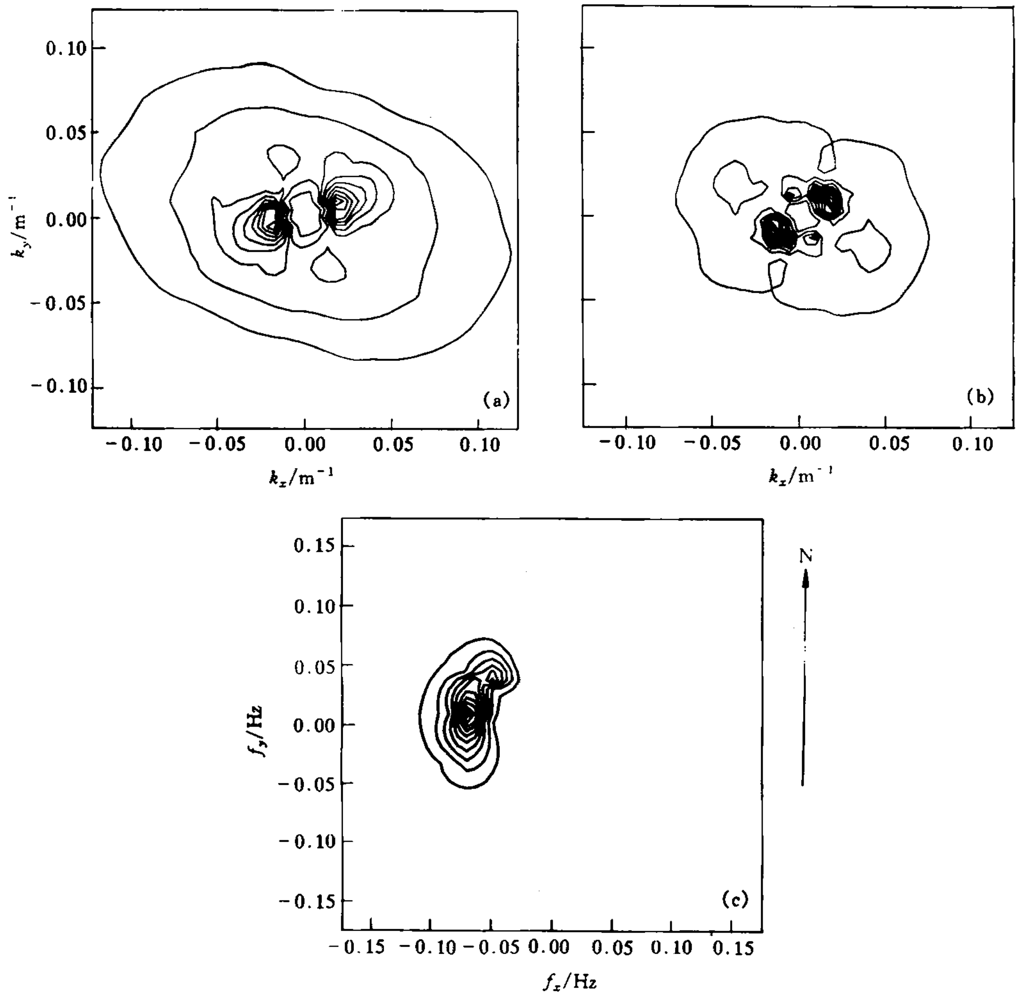



Fig. 3. (a) Stimulation SAR image spectrum; (b) observed SAR image spectrum; (c) inversion of ERS-1 SAR image spectrum.

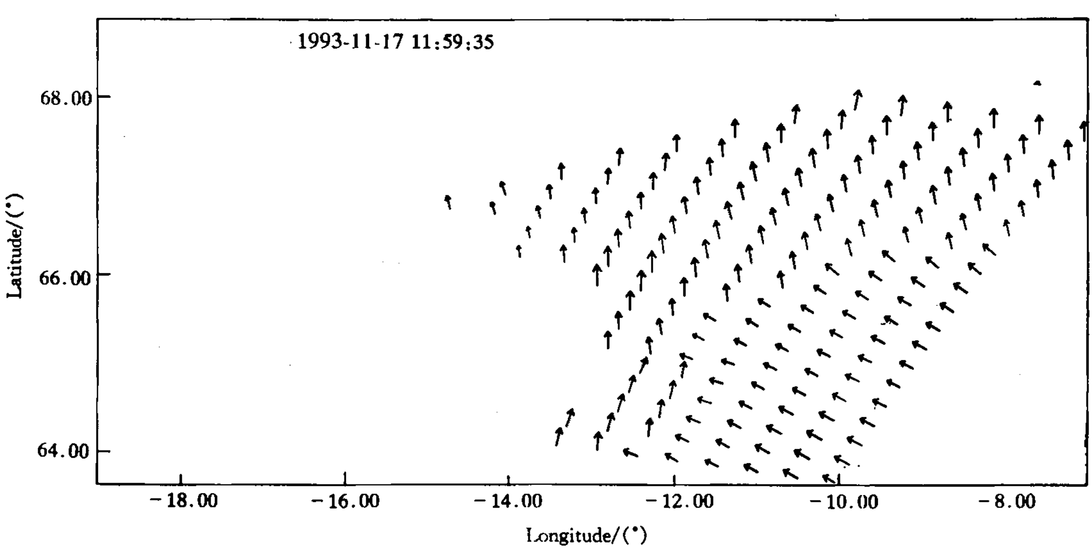



Fig. 4. Wind field derived from ERS-1 scatterometer.

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